Written Calculation Policy

Long Sutton Primary School

POINTS TO REMEMBER: Use the language '**calculation'** not 'sum' ('Sum' means 'plus' or 'total')
Use the language '**digit'** not 'number' ('Number' is the amount or quantity)
Children should be able to use these strategies independently when problem solving before moving on.

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Glossary of Terms

Cardinal number	The number of items in a set, the quantity but not the order of things.
Conservation of number	Understanding that if a group of objects is rearranged, the total number of objects stays the same.
Consecutive	Following in order. Consecutive numbers are adjacent in a count, e.g. 1,2,3,4 are consecutive, and 20,25,30 are consecutive multiples of 5.
Commutativity	For addition and multiplication, numbers in a calculation can be in any order and will result in the same answer. They are commutative. Subtraction and division are not commutative. However children must understand that the numbers in a calculation can still be in any order, but will result in a different answer.
Digit	A symbol of the number system – 0 1 2 3 4 5 6 7 8 9 The position or place of a digit conveys its value.
Dividend	The quantity which is to be divided, e.g. in the calculation 12 ÷ 3, the dividend is 12.
Divisor	The quantity by which another is to be divided, e.g. in the calculation 12 ÷ 3, the divisor is 3.
Long multiplication	A formal calculation strategy that builds on understanding of the grid method into a compact column method. The multiplier is larger than 12 and therefore is partitioned during the process to aid multiplication. Long multiplication is a multi-stage calculation which requires a final addition calculation to reach an answer.
Inverse of multiplication (as a method of division)	Counting up from 0 in multiples to reach a number in order to solve a division calculation. Some children find counting on in the multiples from 0 easier than repeated subtraction, and this is fine as long as they understand that they are using inverse of multiplication, rather than repeated subtraction.
Number line	A line on which numbers are represented by points. Division marks are numbered rather than spaces. They can begin at any number and extend into negative numbers. They can show any number sequence.
Number track	A numbered track along which counters can be moved. The number in a region represents the number of single moves from the start. - Each number occupies a cell and is used to number it. - Numbers may have a matching illustration. - Supports learning to read numbers in numerals. - Supports locating ordered numbers. - They should start at 1 and not 0.
Numeral	A symbol used to denote a number. For example 5, 23 and the Roman V are all numerals.
Ordinal numbers	A term that describes a position within an ordered set, e.g. first, etc.

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Partition	1. To separate a set into subsets. 2. To split a number into component parts. For example, the two-digit number 38 can be partitioned into 30 + 8 or 19 + 9 or 20 + 18, etc.
Principle of Exchange	The naming system when counting collections, that as soon as we have a group of ten we will call them something else. The number we call ten (10 in numerals) is the most important in our naming system, e.g. ten ones are called one ten, ten tens are called one hundred, ten hundreds are called one thousand, etc.
Proportionality	The relationship of one thing to another in terms of size, quantity, or number / out of whole, e.g. 2 out of 5. Proportionality puts the emphasis on the relationship rather than the quantity.
Quotient	The result of a division calculation, e.g. in the calculation 12 ÷ 3, the quotient is 4.
Ratio	The comparison of two properties, e.g. 2:3. All ratio relationships are proportional.
Repeated subtraction	Repeatedly subtracting the same amount each time in order to solve a division calculation. The idea of repeated subtractions should be 'How many times can I take away _____ from _____?'
Representation	The wide variety of ways to capture an abstract mathematical concept or relationship. This may be visible, such as a number sentence, a display of manipulative materials, or a graph, but it may also be an internal way of seeing and thinking about a mathematical idea. Representations can enhance communication, reasoning, and problem-solving abilities; help them make connections among ideas; and aid them in learning new concepts and procedures.
Short multiplication	A formal calculation strategy that builds on understanding of the grid method into a compact column method. The multiplier is 12 or less and therefore is not partitioned during the process as the calculations should rely on knowledge of multiplication facts up to 12x12. An expanded short multiplication method details each stage in brackets and shows clear connections to the grid method, which will bridge understanding between this and the grid method.
Subitising	This is the process whereby we recognise the size of a set, its cardinality, from the pattern or structure without having to count the number of objects. For example recognising that there are five dots in a pattern (on a dice).
Zero	1. Nought or nothing. 2. In a place-value system, a place holder, e.g. 105. 3. The cardinal number of an empty set.

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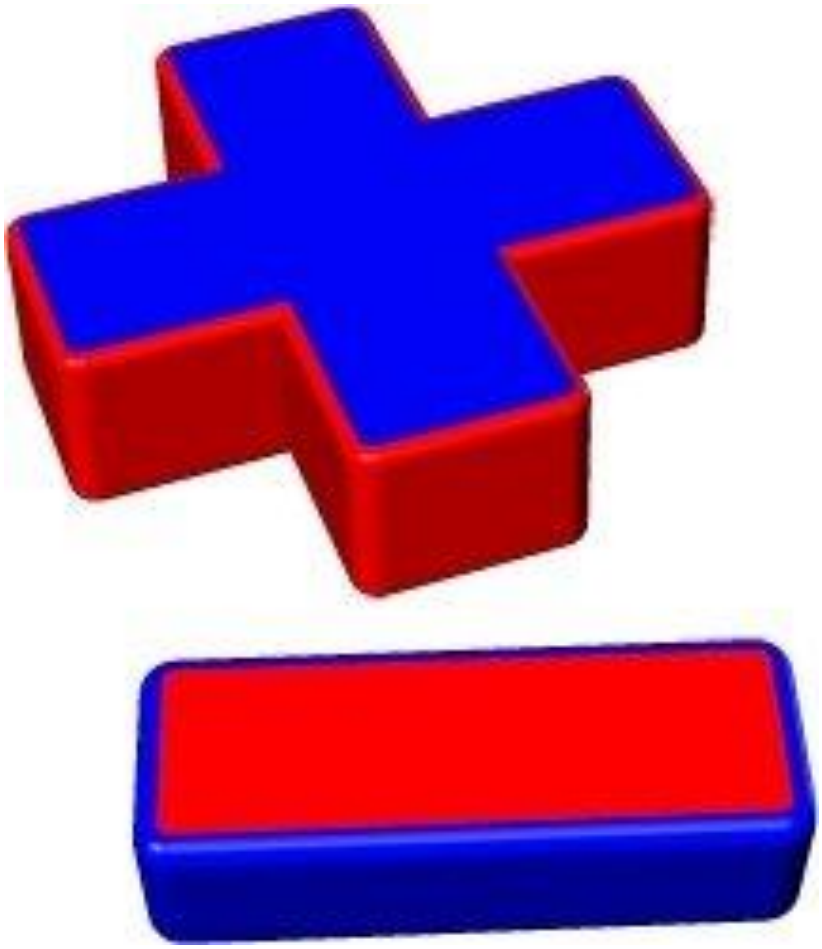
Addition and Subtraction



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Stage 1 Addition

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

Add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundred boundary, units/ones boundary, inverse, how many more to make…?, is the same as,

Children will use practical equipment to combine groups of objects to find a total. Practical resources will support children's development of mental pictures and images that they will be able to use through later stages of understanding.

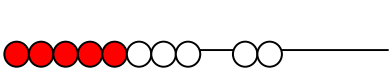
Children will begin to understand **commutativity** and **the principle of exchange**. They will be confident in using the terms 'worth' and 'value' when talking about single-digit numbers.

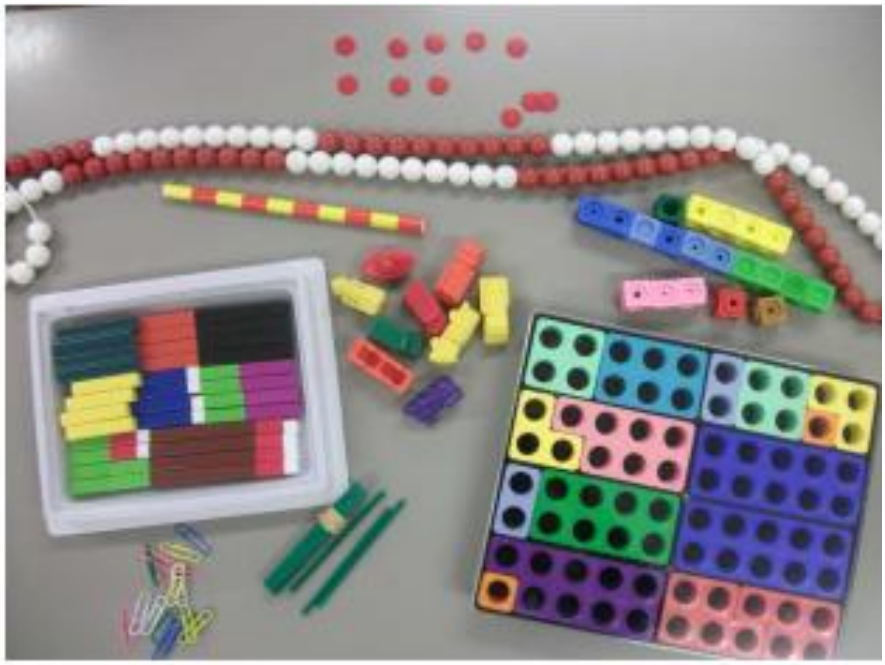
[The Principle of Exchange can be taught through activities such as 'swaps', i.e. "I'll swap you one lorry for three cars." This will extend into: "One lorry is worth three cars." This will be used when extending addition to using Deans cubes, e.g. ten ones are worth one ten, etc.]

Children can represent calculations using objects and talk about their representations.

Children should understand **conservation of number** (that the number of objects does not change when the objects are rearranged, i.e. they will not need to recount them).

Use beads, counters and other objects (including Cuisenaire rods and Numicon)





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Stage 1 Subtraction

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

Subtract, subtraction, take away, minus, decrease, leave, how many are left/left over? difference between, half, halve, how many more than/feweris…? how much more/less is…? Is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

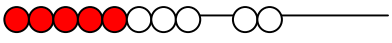
Children will use practical equipment to physically remove an amount from the group to find the total remaining.

Practical resources will support children's development of mental pictures and images that they will be able to use through later stages of understanding.

Children can represent calculations using objects and talk about their representations.

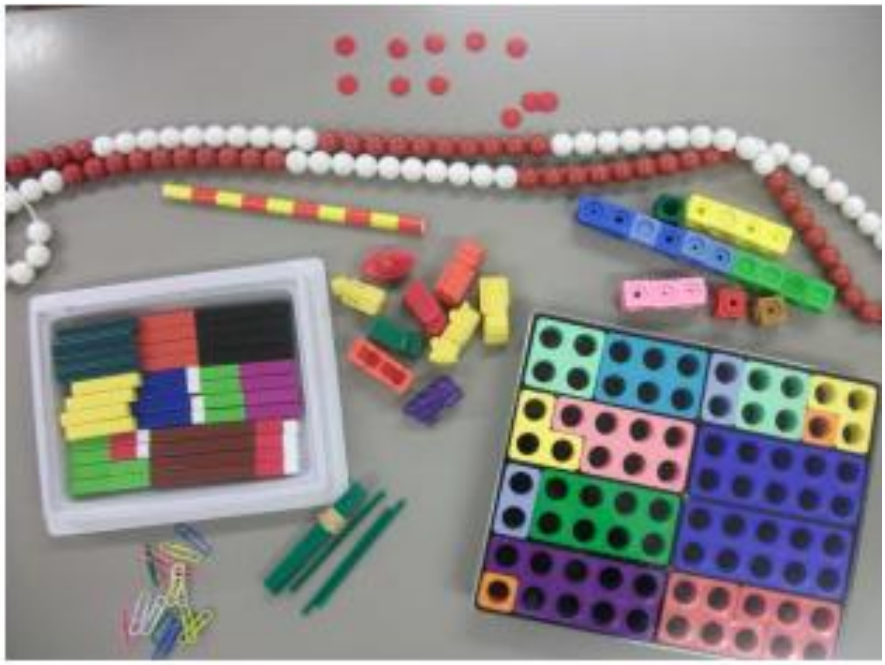
Children should understand **conservation of number** (that the number of objects does not change when the objects are rearranged, i.e. they will not need to recount them).

Use beads, counters and other objects (including Cuisenaire rods and Numicon)



2+ 8=10 8+2=10
10-8=2 10-8=2

Counters



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Stage 2 Addition

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

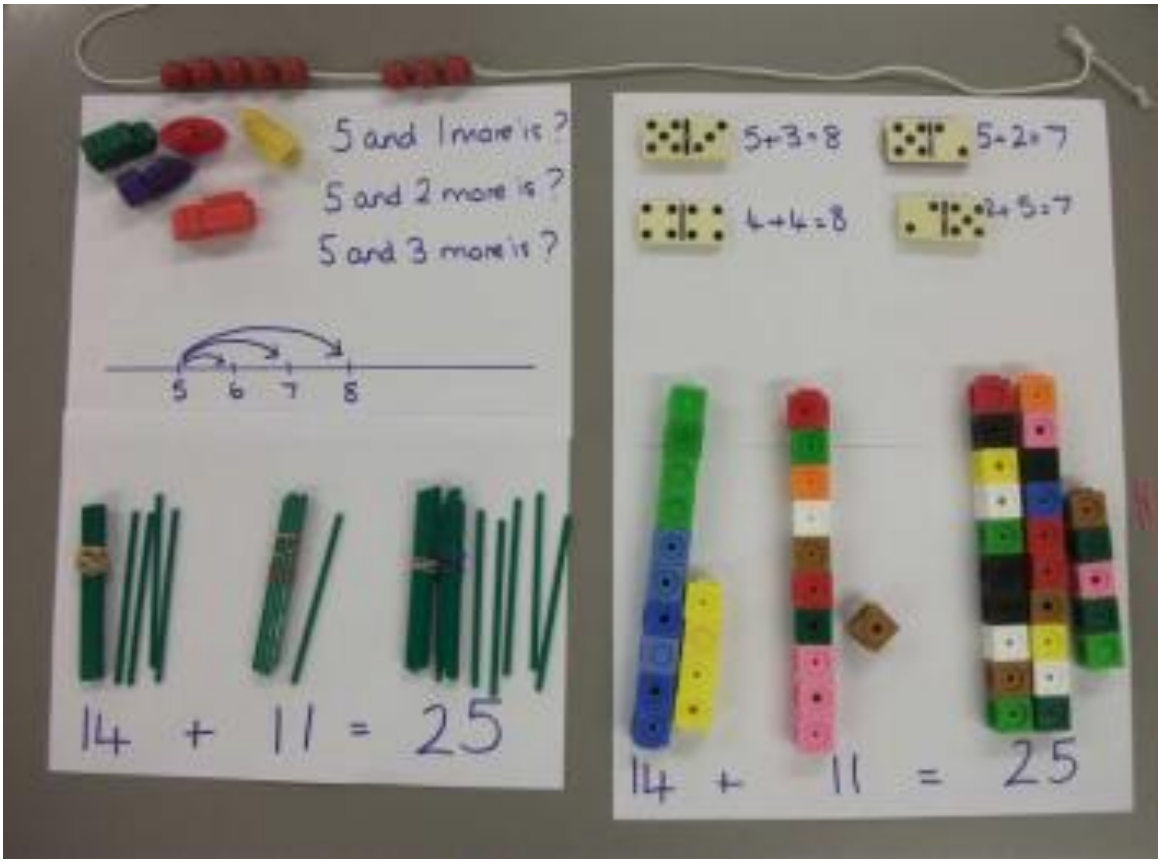
Add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundred boundary, units/ones boundary, inverse, how many more to make…?, is the same as,

Practical resources will continue to support children's development of mental pictures and images. As these become firm, children will begin to develop ways to represent their mental images and their practical resources using pictures.

The children will begin to use number sentences alongside their pictures and practical resources.

They will also begin to think and talk flexibly about addition and its inverse of subtraction (using a range of vocabulary and different real life situations).

The direct link between addition and subtraction should be made explicit when using models and representations.



Other objects can be used as appropriate.

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Stage 2 Subtraction

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

Subtract, subtraction, take away, minus, decrease, leave, how many are left/left over? difference between, half, halve, how many more than/feweris…? how much more/less is…? Is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

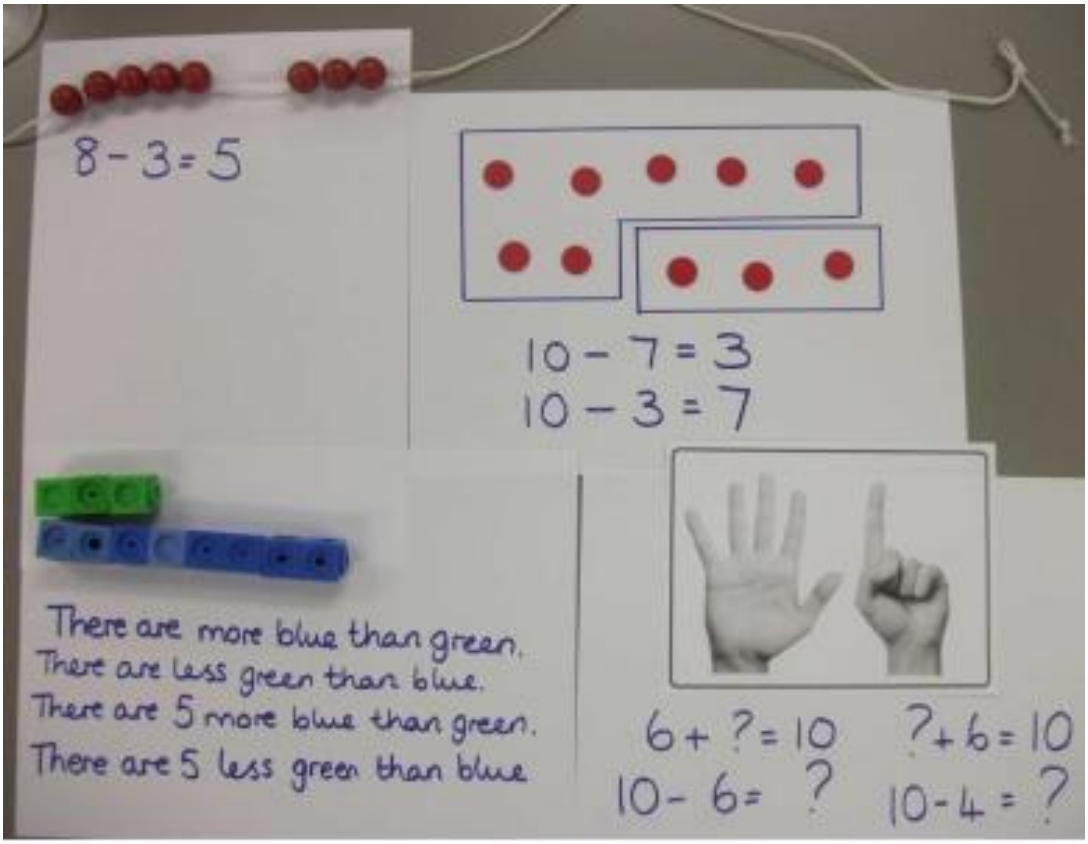
Practical resources will continue to support children's development of mental pictures and images. As these become firm, children will begin to develop ways to represent their mental images and their practical resources using pictures.

The children will begin to use number sentences alongside their pictures and practical resources.

They will also begin to think and talk flexibly about subtraction and its inverse of addition (using a range of vocabulary and different real life situations).

Children will understand that subtraction is not commutative and so the numbers in a calculation can be in any order, but will result in a different answer.

The direct link between addition and subtraction should be made explicit when using models and representations.



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Stage 3 Addition

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

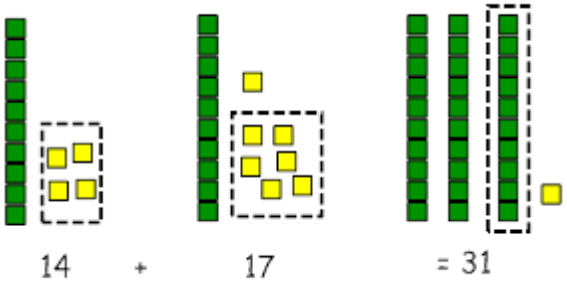
Add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundred boundary, units/ones boundary, inverse, how many more to make…?, is the same as,

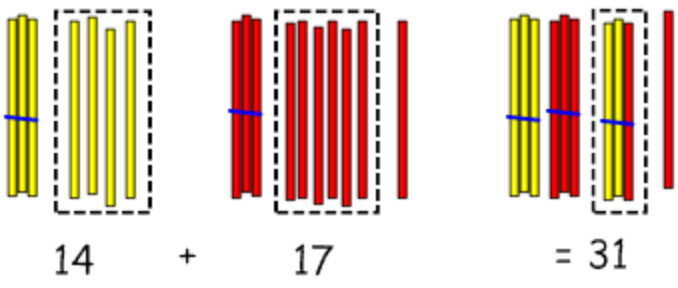
Children will now be confident in using concrete equipment to help them combine groups of objects with numbers up to 20.

They will continue using practical equipment, alongside new resources such as number tracks, number lines and hundred squares to support their mental methods.

Children will start to work with totals greater than 20 which require them to apply their knowledge of the **principle of exchange** (i.e. using straws, Deans cubes, etc). They will talk confidently about this and be able to explain what they are doing and why.

14 + 17 =





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Stage 3 Subtraction

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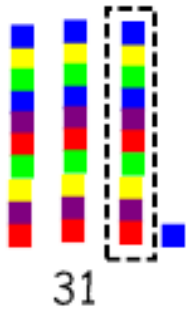
Subtract, subtraction, take away, minus, decrease, leave, how many are left/left over? difference between, half, halve, how many more than/feweris…? how much more/less is…? Is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

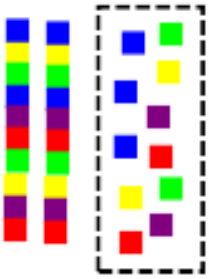
Children will now be confident in using concrete equipment to help them 'take away' and 'find the difference'.

They will continue using practical equipment, alongside new resources such as number tracks, number lines and hundred squares to support their mental methods.

Children will start to work with totals greater than 20 which require them to apply their knowledge of the **principle of exchange** (i.e. using straws, Deans cubes, etc). They will talk confidently about this and be able to explain what they are doing and why.

31 – 14 =

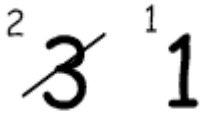



31 is repartitioned into 20 and 11 using the principle of exchange.


14 can now be removed from the 31, leaving 17.

Any resources (including Dienes cubes and straws) can be used in this way.

As children become used to repartitioning numbers, they can be introduced to formal notation of the repartitioning.



*This is now 20 and 11.

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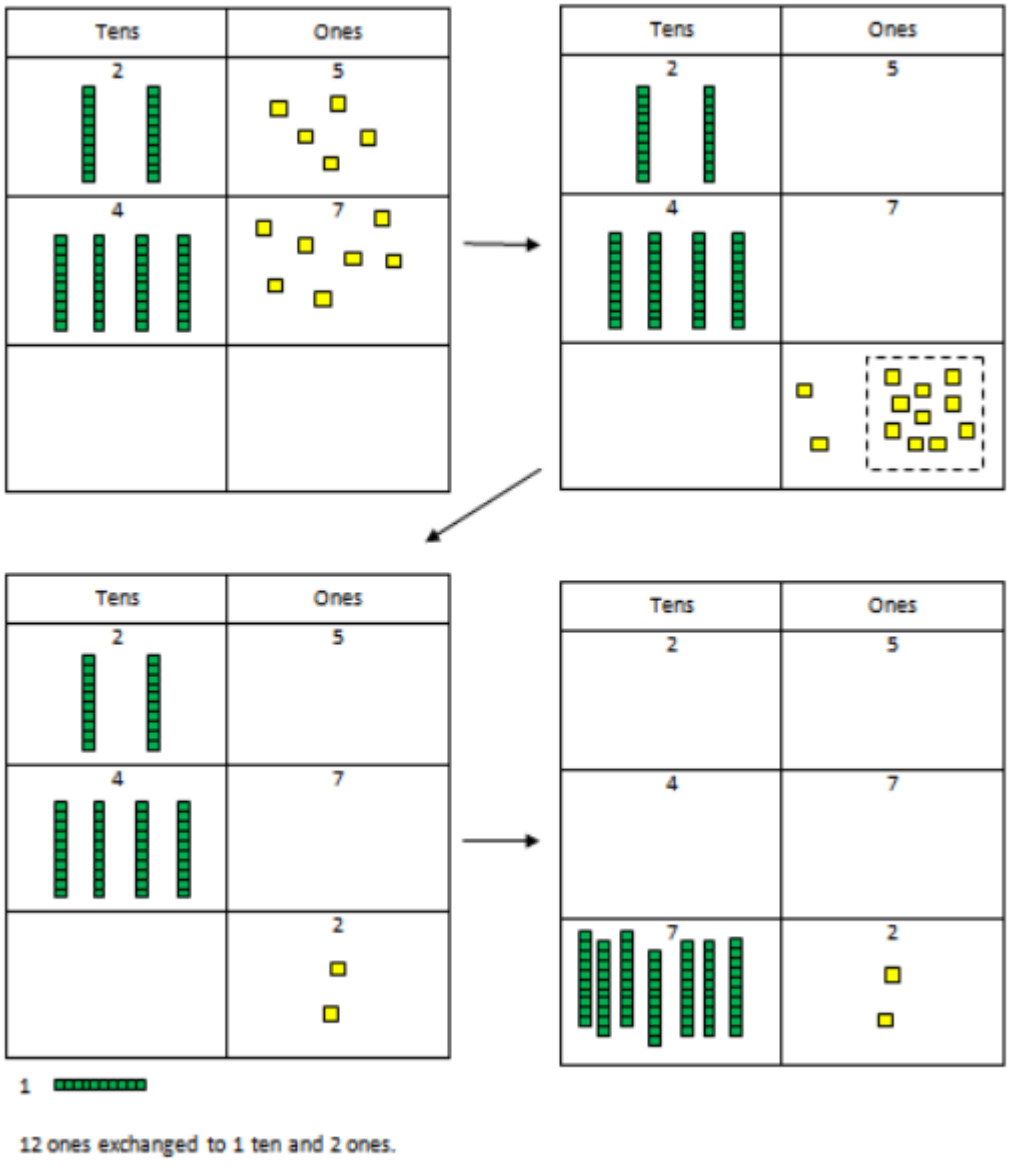
Stage 4 Addition

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

Add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundred boundary, units/ones boundary, inverse, how many more to make…?, is the same as,

Children will now be confident in using concrete equipment to combine objects using the **principle of exchange**.

They will now begin to organise their concrete equipment (e.g. Straws, Dienes, Place Value Counters) in a vertical manner, where their combined totals are situated at the bottom.



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Stage 4 Subtraction

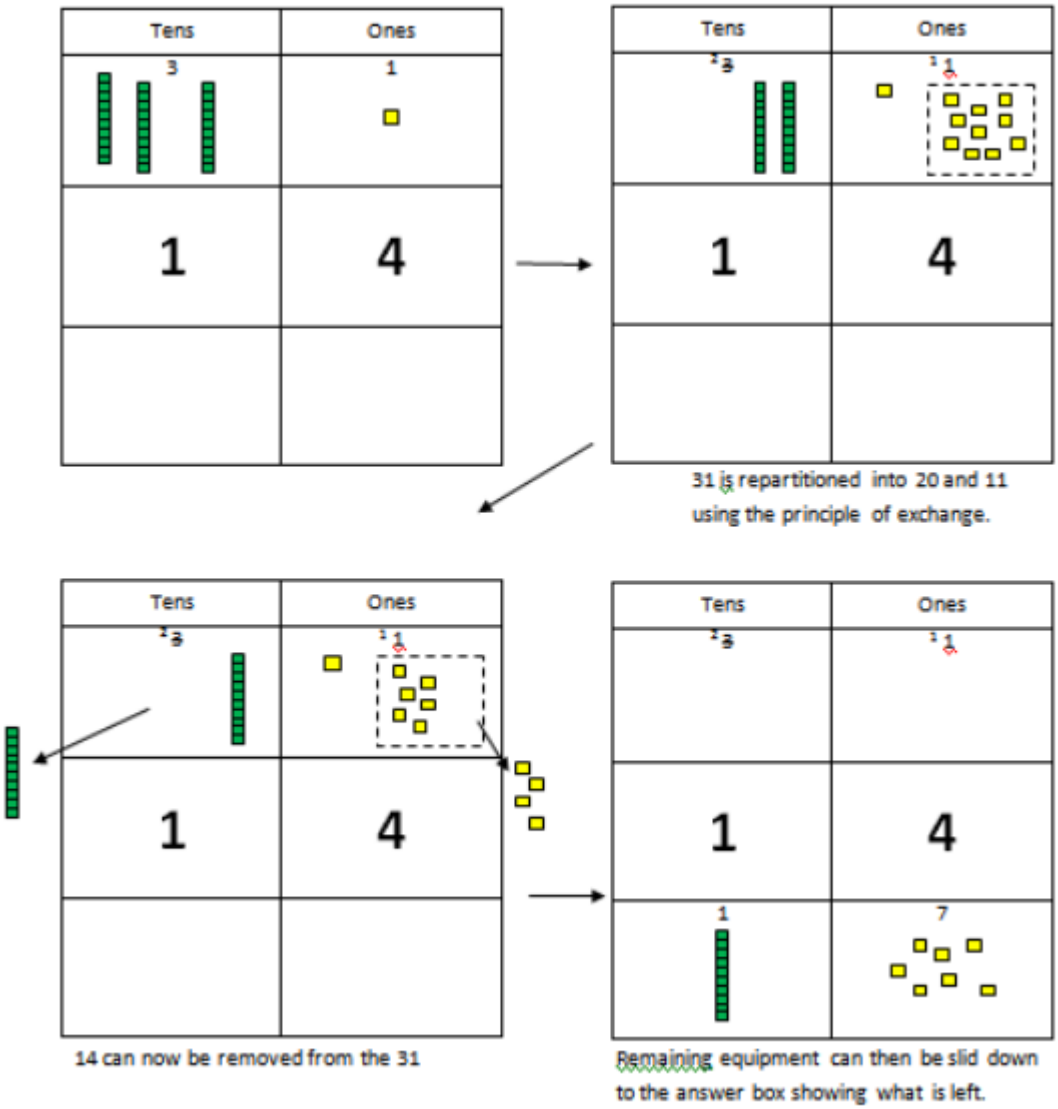
Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

Subtract, subtraction, take away, minus, decrease, leave, how many are left/left over? difference between, half, halve, how many more than/feweris…? how much more/less is…? Is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Children are now confident in using concrete equipment to help them 'take away' and 'find the difference' using the **principle of exchange** appropriately.

They will now begin to organise their concrete equipment (e.g. Straws, Dienes, Place Value Counters) in a vertical manner, where their combined totals are situated at the bottom.

31 - 14



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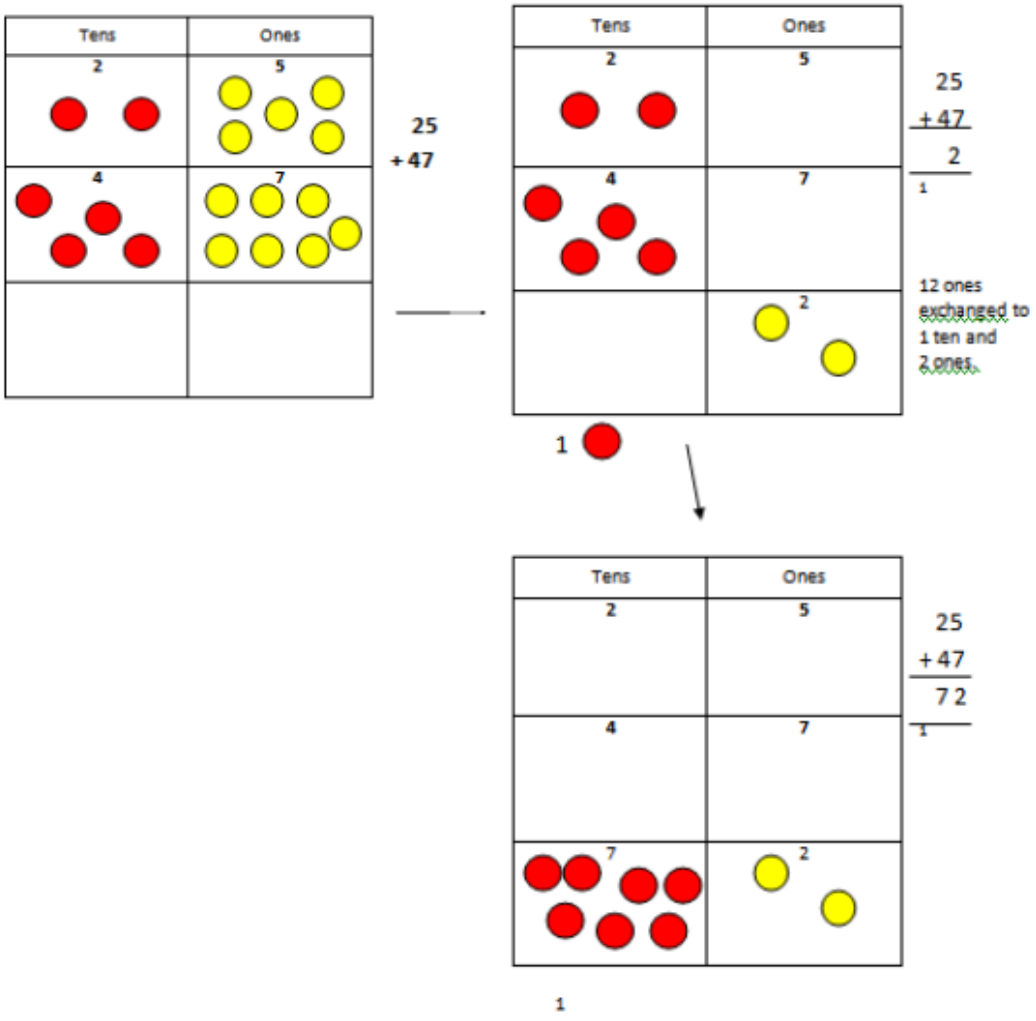
Stage 5 Addition

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

Add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundred boundary, units/ones boundary, inverse, how many more to make…?, is the same as,

Children will now be secure in organising their concrete equipment in a vertical manner where their combined totals are situated at the bottom.

They will now be able to make the links between this representation and the formal column addition when seen alongside each other.



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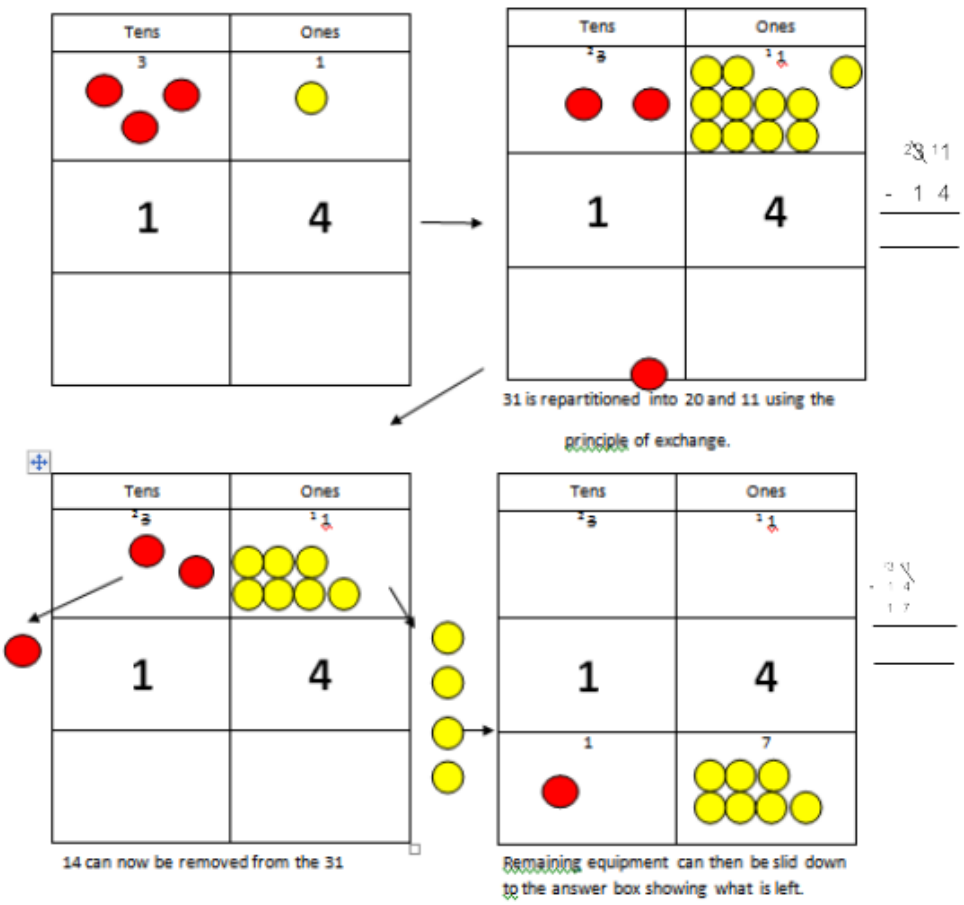
Stage 5 Subtraction

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):
Subtract, subtraction, take away, minus, decrease, leave, how many are left/left over? difference between, half, halve, how many more than/feweris…? how much more/less is…? Is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Children will now be secure in organising their concrete equipment in a vertical manner for subtraction using the **principle of exchange** appropriately.

They will now be able to make the links between this representation and the formal column subtraction when seen alongside each other.



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Stage 6 Addition

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

Add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundred boundary, units/ones boundary, inverse, how many more to make…?, is the same as,

Children will have a full understanding of the links between the concrete representation for column addition and the formal written method.

They will now be able to explore calculating with larger numbers using their understanding of the formal written method.

$$\begin{array}{r} 3\ 2\ 7 \\ +\ 4\ 9\ 6 \\ \hline 8\ 2\ 3 \\ \hline 1\ \ 1 \end{array}$$

Calculating with decimals

When working with decimals, the above stages should always be followed to allow for the development of conceptual understanding. The use of concrete equipment is essential at these stages to secure understanding of the value of each digit in a number (e.g. Place Value Counters, Money). Wherever possible, decimal calculations should be linked to real-life experiences, e.g. money and measures.

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Stage 6 Subtraction

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

Subtract, subtraction, take away, minus, decrease, leave, how many are left/left over? difference between, half, halve, how many more than/feweris…? how much more/less is…? Is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Children will have a full understanding of the links between the concrete representation for column subtraction and the formal written method.

They will now be able to explore calculating with larger numbers using their understanding of the formal written method.

$$\begin{array}{r} 7\ {}^{7}\cancel{8}\ {}^{1}4 \\ -\ 2\ 5\ 9 \\ \hline 5\ 2\ 5 \\ \hline \end{array}$$

Calculating with decimals

When working with decimals, the above stages should always be followed to allow for the development of conceptual understanding. The use of concrete equipment is essential at these stages to secure understanding of the value of each digit in a number (e.g. Place Value Counters, Money). Wherever possible, decimal calculations should be linked to real-life experiences, e.g. money and measures.

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Multiplication and Division

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Stage 1 Multiplication

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

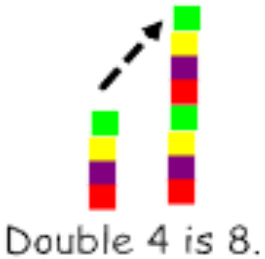
counting, steps, each, doubling, scaling, times, twice as big, _____ as big, count in ones, count in ____, lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times, … times, ten times……, times as (big, long, wide…and so on), repeated addition, array, row, column, double, group in pairs, threes…tens, equal groups of, multiplication, product, inverse.

Children will experience practical opportunities involving equal sets or groups using a wide variety of equipment. Practical resources will support children's development of mental pictures and images.

Children will begin to orally count in different multiples including **twos, fives and tens**, making links to natural groupings (e.g. pairs of socks, legs on animals) and the practical resources used.

Children can begin to recognise and continue patterns of multiples using a range of practical resources, e.g. threading beads with two of each colour.

They will begin to use the language and associated representations of doubling.



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Stage 1 Division

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

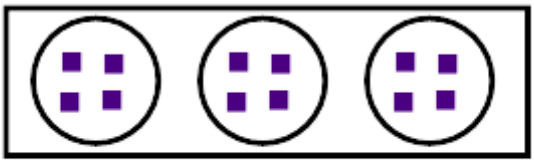
halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ___, ___equal groups

Children will explore the language of sharing. Children will experience practical activities in 'sharing' objects between a small number of groups/people with the emphasis on sharing equally.

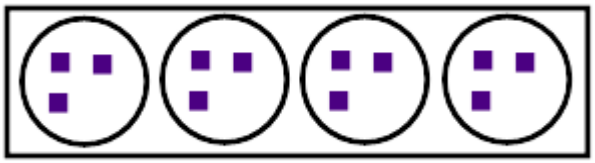
Alongside this, with equal weighting, children should be introduced to 'grouping' objects as a representation of division (e.g. 'each person gets 2') with the emphasis on equal groups.

They will begin to use the language and associated representations of halving.

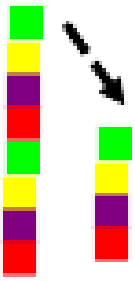
Children can be encouraged to develop ways of recording their findings using pictures.



12 shared into 3 equal group.
12 shared equally into groups of 4.



12 shared into 4 equal groups.
12 shared equally into groups of 3.



Half of 8 is 4.

POINTS TO REMEMBER: Use the language '**calculation'** not 'sum' ('Sum' means 'plus' or 'total')
Use the language '**digit'** not 'number' ('Number' is the amount or quantity)
Children should be able to use these strategies independently when problem solving before moving on.

Stage 2 Multiplication

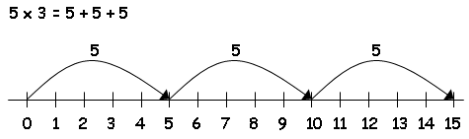
Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

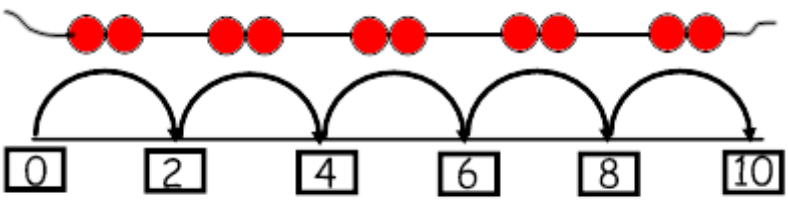
counting, steps, each, doubling, scaling, times, twice as big, _____ as big, count in ones, count in ____, lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times, … times, ten times……, times as (big, long, wide…and so on), repeated addition, array, row, column, double, group in pairs, threes…tens, equal groups of, multiplication, product, inverse.

Children will begin to arrange objects in equal groups to aid counting:

They will continue to count in multiples and begin to relate this to multiplication through finger counting.

Children will be introduced to a variety of representations of repeated addition; they will see the representations alongside each other and begin to make connections between them.



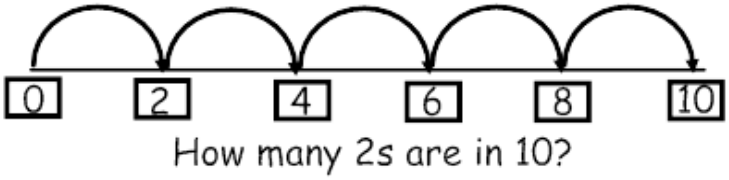


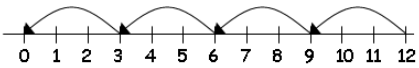
POINTS TO REMEMBER: Use the language '**calculation'** not 'sum' ('Sum' means 'plus' or 'total')
Use the language '**digit'** not 'number' ('Number' is the amount or quantity)
Children should be able to use these strategies independently when problem solving before moving on.

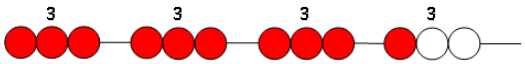
Stage 2 Division

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):
halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ___, ___equal groups

Children will relate the grouping of objects to repeated subtraction and begin to represent this using a number line whilst continuing to use concrete equipment.



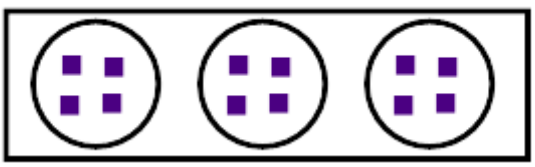


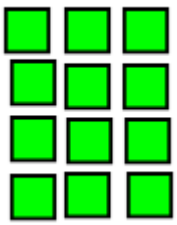


The bead bar will help children with interpreting division calculations such as 10 ÷ 5 as 'how many 5s make 10?'

Children will use their knowledge of counting up in multiples to solve division calculations and recognise that this is the **inverse of multiplication**.

Children will continue to group and share equally using concrete equipment and will now begin to organise their groups into an array rather than scattered groupings.





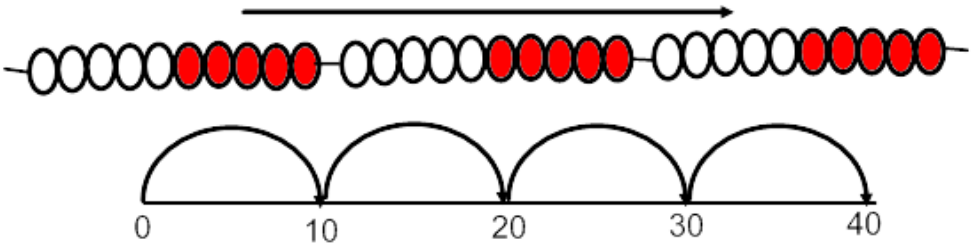
POINTS TO REMEMBER: Use the language '**calculation'** not 'sum' ('Sum' means 'plus' or 'total')
Use the language '**digit'** not 'number' ('Number' is the amount or quantity)
Children should be able to use these strategies independently when problem solving before moving on.

Stage 2 Multiplication (cont.)

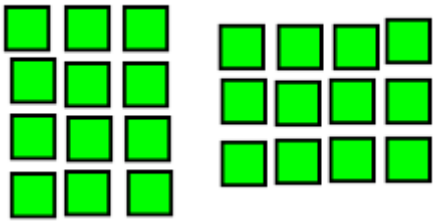
Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

counting, steps, each, doubling, scaling, times, twice as big, _____ as big, count in ones, count in ____, lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times, … times, ten times……, times as (big, long, wide…and so on), repeated addition, array, row, column, double, group in pairs, threes…tens, equal groups of, multiplication, product, inverse.

Count in tens from zero



Children will be introduced to the array, using concrete equipment, for small numbers as a way of organising groups to show repeated addition and commutativity. They should explore arrays in the world around us, e.g. egg boxes, baking trays, wrapping papers; and use them to answer questions such as "How many eggs would we need to fill the egg box?" "How do you know?"



POINTS TO REMEMBER: Use the language '**calculation'** not 'sum' ('Sum' means 'plus' or 'total')
Use the language '**digit'** not 'number' ('Number' is the amount or quantity)
Children should be able to use these strategies independently when problem solving before moving on.

Stage 2 Division (cont.)

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ___, ___equal groups

The direct link between multiplication and division should be made explicit when using models and representations.

Children will continue to make links between division and fractions. They will be aware that the division sign is the equivalent to the fractions line, and p ÷ q can be written as $\frac{p}{q}$

$1 \div 2$

$\frac{1}{2}$

POINTS TO REMEMBER: Use the language '**calculation'** not 'sum' ('Sum' means 'plus' or 'total')
Use the language '**digit'** not 'number' ('Number' is the amount or quantity)
Children should be able to use these strategies independently when problem solving before moving on.

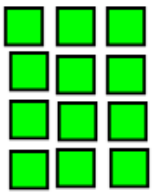
Stage 3 Multiplication

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

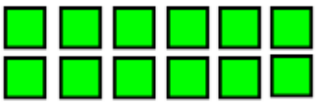
counting, steps, each, doubling, scaling, times, twice as big, _____ as big, count in ones, count in ____, lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times, … times, ten times……, times as (big, long, wide…and so on), repeated addition, array, row, column, double, group in pairs, threes…tens, equal groups of, multiplication, product, inverse.

Children will continue to count in multiples and begin to relate this to multiplication through finger counting and using counting sticks.

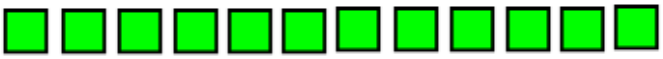
They will be able to model a calculation using a practical array which demonstrates an effective method of counting and the link to repeated addition. Children need to explore related multiplication facts of a given number by making a variety of arrays and explain what they show.



3 x 4 = 12
4 x 3 = 12
3 + 3 + 3 + 3 = 12
4 + 4 + 4 = 12

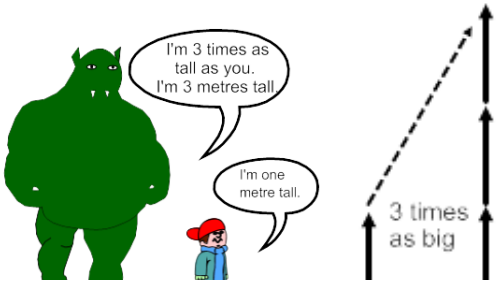


2 x 6 = 12
6 x 2 = 12
6 + 6 = 12
2 + 2 + 2 + 2 + 2 + 2 = 12



1 x 12 = 12
12 x 1 = 12

The children should be confident with their use of the language of scaling when talking about multiplication.



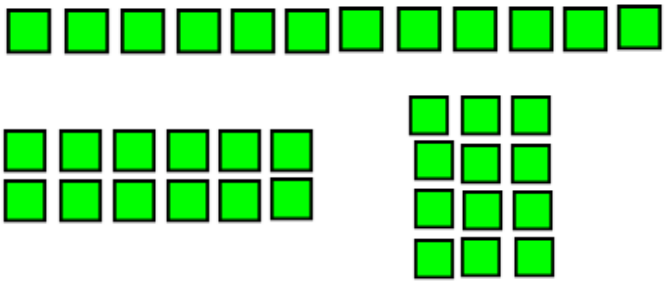
POINTS TO REMEMBER: Use the language '**calculation'** not 'sum' ('Sum' means 'plus' or 'total')
Use the language '**digit'** not 'number' ('Number' is the amount or quantity)
Children should be able to use these strategies independently when problem solving before moving on.

Stage 3 Division

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):
halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ___, ___equal groups

Children will continue to use their knowledge of counting in multiples to support the inverse of multiplication and repeated subtraction.

Children will build on their use of concrete arrays for division recognising the links to repeated subtraction and the inverse of multiplication in order to derive the associated division facts. Children need to explore related division facts of a given number by making a variety of arrays and explaining what they show.



12 into ____ equal groups gives ____ in each group.
12 into equal groups of _____ gives ____ groups.

The children should be confident with their use of the language of scaling when talking about division with links made to simple fractions (e.g. half the size, three times smaller).

POINTS TO REMEMBER: Use the language '**calculation'** not 'sum' ('Sum' means 'plus' or 'total')
Use the language '**digit'** not 'number' ('Number' is the amount or quantity)
Children should be able to use these strategies independently when problem solving before moving on.

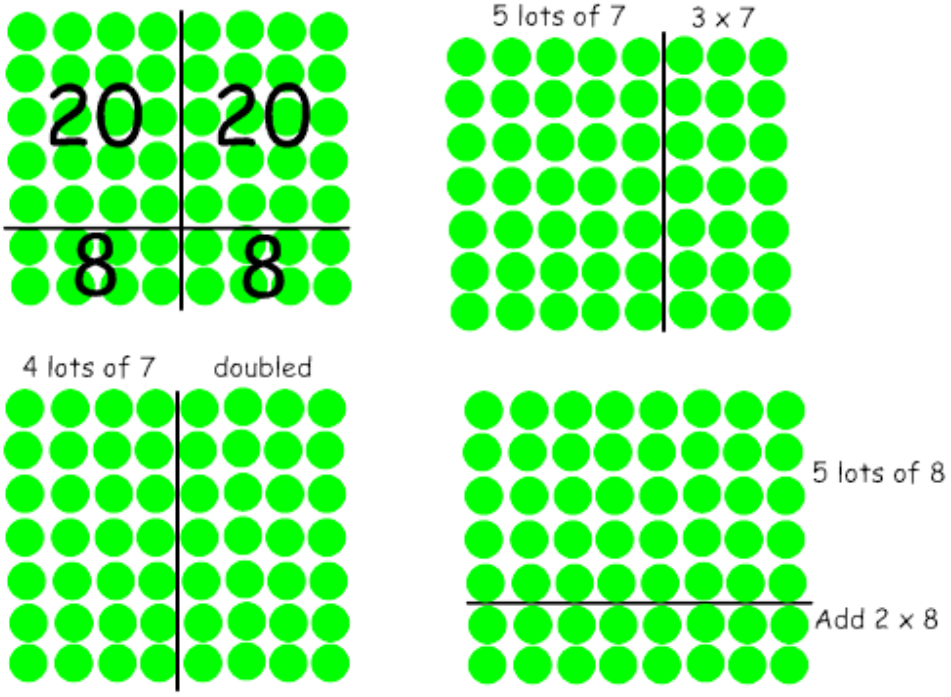
Stage 4 Multiplication

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

counting, steps, each, doubling, scaling, times, twice as big, _____ as big, count in ones, count in ____, lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times, … times, ten times……, times as (big, long, wide…and so on), repeated addition, array, row, column, double, group in pairs, threes…tens, equal groups of, multiplication, product, inverse.

Children will explore practical arrays for larger numbers. They will think flexibly when working with arrays and will be encouraged to look at arrays beyond repeated addition. They will look for 'friendly' numbers to help them efficiently calculate totals within arrays. E.g. for 7x8… Children may find counting in 7s or 8s tricky but they can look for 'friendly' numbers which are easier to calculate, e.g. 4x5, 4x2, 4x5, 4x2.

Thinking flexibly about 7 x 8



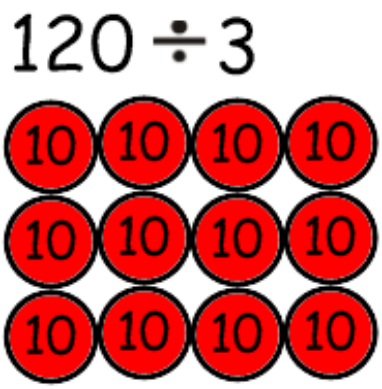
Children should continue to experience the language of scaling (e.g. scaling up pictures by multiplying by powers of 10, or multiplying by powers of 1000 in converting between units of measure).

POINTS TO REMEMBER: Use the language '**calculation'** not 'sum' ('Sum' means 'plus' or 'total')
Use the language '**digit'** not 'number' ('Number' is the amount or quantity)
Children should be able to use these strategies independently when problem solving before moving on.

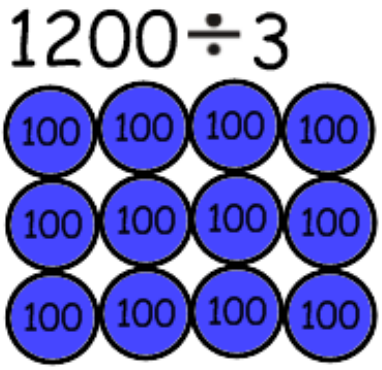
Stage 4 Division

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):
halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ___, ___equal groups

Children will continue to organise groups into an array now working with larger numbers by either grouping or sharing. Children will be able to explain all the facts they know about a given array with no remainder. They should be making arrays with the equipment to establish 'How many in each group?' or 'How many groups?' Children should continue to experience the language of scaling (e.g. scaling down pictures by dividing by powers of 10, or dividing by powers of 1000 in converting between units of measure).



120 shared equally between 3 is 40.
120 shared equally between 4 is 30.
3 equal groups of 40 make 120.
4 equal groups of 30 make 120.



1200 shared equally between 3 is 400.
1200 shared equally between 4 is 300.
3 equal groups of 400 make 1200.
4 equal groups of 300 make 1200.

POINTS TO REMEMBER: Use the language '**calculation'** not 'sum' ('Sum' means 'plus' or 'total')
Use the language '**digit'** not 'number' ('Number' is the amount or quantity)
Children should be able to use these strategies independently when problem solving before moving on.

Stage 5 Multiplication

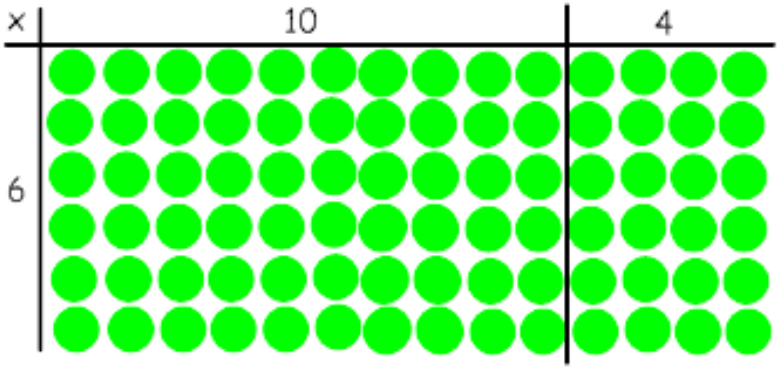
Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

counting, steps, each, doubling, scaling, times, twice as big, _____ as big, count in ones, count in ____, lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times, … times, ten times……, times as (big, long, wide…and so on), repeated addition, array, row, column, double, group in pairs, threes…tens, equal groups of, multiplication, product, inverse.

Children will continue to work with arrays, exploring larger numbers, leading into the grid method of multiplication. Practical experiences may still be required for some children as they enter this stage. To begin with, children should see the array with the grid lines. When appropriate, children should move to using the grid displaying the numbers only.

Children should begin using grid method for 2- and 3- digit by 1 digit numbers and should be given the chance to relate this to facts they know about arrays where needed.

Throughout this stage, children should be encouraged to estimate an approximate answer in order to check for reasonableness and this should become standard practice.



$(6 \times 10) + (6 \times 4)$
$60 + 24$
$= 84$

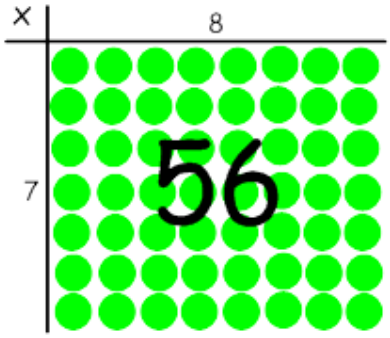
×	10	4
6	60	24

POINTS TO REMEMBER: Use the language '**calculation'** not 'sum' ('Sum' means 'plus' or 'total')
Use the language '**digit'** not 'number' ('Number' is the amount or quantity)
Children should be able to use these strategies independently when problem solving before moving on.

Stage 5 Division

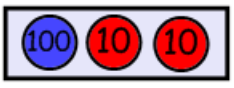
Vocabulary (Ensure the correct vocabulary is used at all stages of learning):
halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ___, ___equal groups

Children will continue to work with concrete arrays, exploring known multiplication/division facts, with the use of grid lines to begin to make the link to short division where numbers are easily divisible. The children understand that the array within short division can be interpreted for either 'sharing between' or 'equal groups of' where the dots within the array each represent 1.

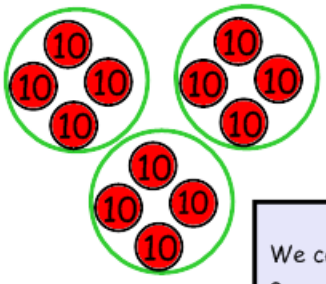


How many equal groups of 7 can I make?
(grouping is represented in the columns)
or
If I put these into 7 equal groups,
how many in each group?
(sharing between is represented in the rows).

Children will begin to use counters within an array to show the sharing model of division, using their knowledge of the principle of exchange where necessary. At this stage, children are encouraged to consider the links between the sharing model and fractions.

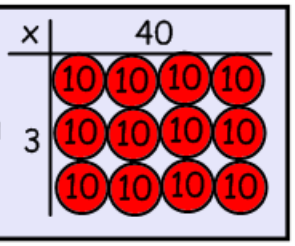


120 can be exchanged for 12 tens in order to make an array.



120 shared into 3 equal groups
gives 40 in each group.

We can explicitly see 40 three times;
3 rows of 40, 1/3 of 120 is 40.
We can divide the array into three, and there is 40 in each part.



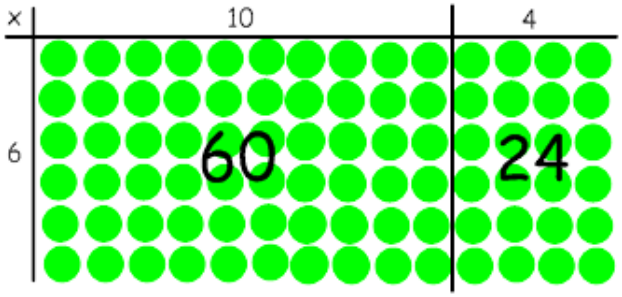
POINTS TO REMEMBER: Use the language '**calculation'** not 'sum' ('Sum' means 'plus' or 'total')
Use the language '**digit'** not 'number' ('Number' is the amount or quantity)
Children should be able to use these strategies independently when problem solving before moving on.

Stage 6 Multiplication

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

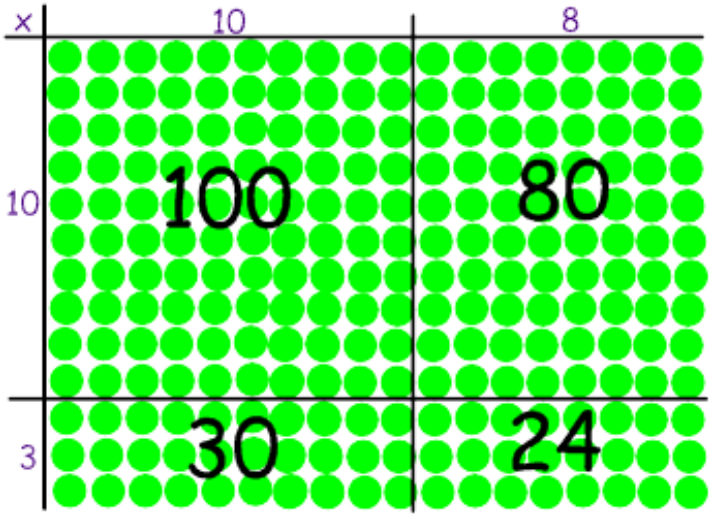
counting, steps, each, doubling, scaling, times, twice as big, _____ as big, count in ones, count in ____, lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times, … times, ten times……, times as (big, long, wide…and so on), repeated addition, array, row, column, double, group in pairs, threes…tens, equal groups of, multiplication, product, inverse.

Children will now be secure in using the grid method for multiplying by one-digit numbers and will begin to explore the links between the grid method and the expanded method of short multiplication.



$$\begin{array}{r} 1\ 4 \\ \times\quad 6 \\ \hline 2\ 4 \quad (6\times4) \\ +\ 6\ 0 \quad (6\times10) \\ \hline 8\ 4 \end{array}$$

Children will also begin to explore the use of arrays and the grid method for multiplying by two-digit numbers.



$$\begin{array}{r} 180 \\ +\ 54 \\ \hline 234 \\ \hline 1 \end{array}$$

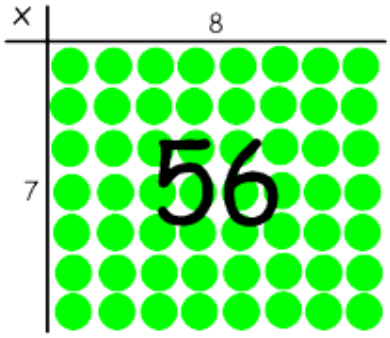
x	10	8
10	100	80
3	30	24

POINTS TO REMEMBER: Use the language '**calculation'** not 'sum' ('Sum' means 'plus' or 'total')
Use the language '**digit'** not 'number' ('Number' is the amount or quantity)
Children should be able to use these strategies independently when problem solving before moving on.

Stage 6 Division

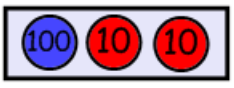
Vocabulary (Ensure the correct vocabulary is used at all stages of learning):
halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ___, ___equal groups

Children will continue to work with concrete arrays, exploring known multiplication/division facts, with the use of grid lines to begin to make the link to short division where numbers are easily divisible. The children understand that the array within short division can be interpreted for either 'sharing between' or 'equal groups of' where the dots within the array each represent 1.

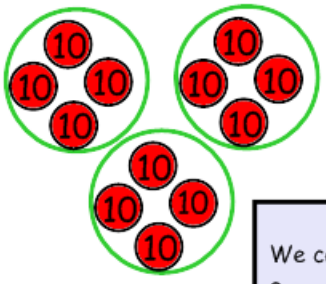


How many equal groups of 7 can I make?
(grouping is represented in the columns)
or
If I put these into 7 equal groups,
how many in each group?
(sharing between is represented in the rows).

Children will begin to use counters within an array to show the sharing model of division, using their knowledge of the principle of exchange where necessary. At this stage, children are encouraged to consider the links between the sharing model and fractions.

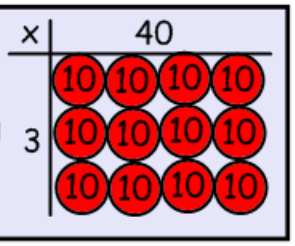


120 can be exchanged for 12 tens in order to make an array.



120 shared into 3 equal groups
gives 40 in each group.

We can explicitly see 40 three times;
3 rows of 40, 1/3 of 120 is 40.
We can divide the array into three, and there is 40 in each part.



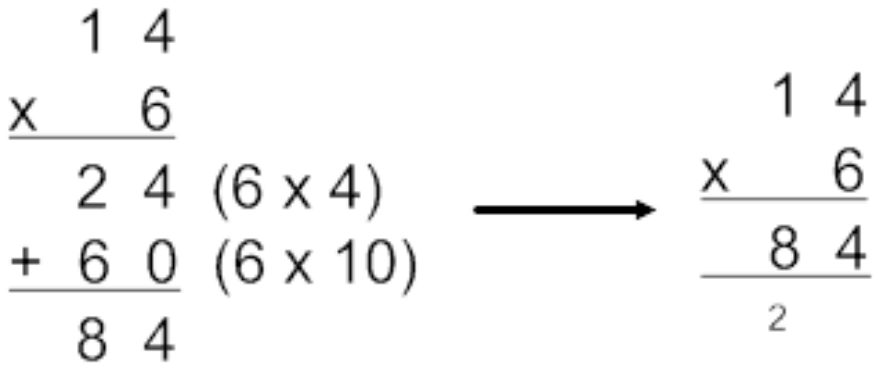
POINTS TO REMEMBER: Use the language '**calculation'** not 'sum' ('Sum' means 'plus' or 'total')
Use the language '**digit'** not 'number' ('Number' is the amount or quantity)
Children should be able to use these strategies independently when problem solving before moving on.

Stage 7 Multiplication

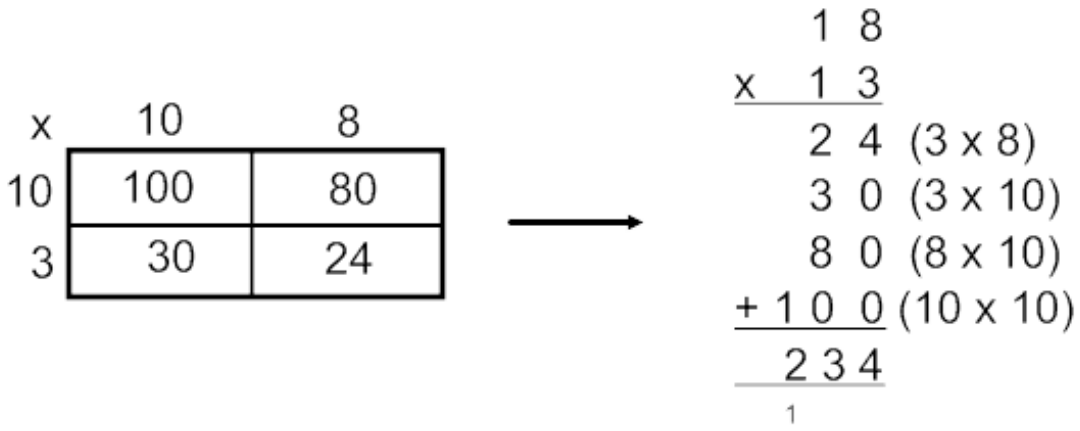
Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

counting, steps, each, doubling, scaling, times, twice as big, _____ as big, count in ones, count in ____, lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times, … times, ten times……, times as (big, long, wide…and so on), repeated addition, array, row, column, double, group in pairs, threes…tens, equal groups of, multiplication, product, inverse.

Children will now have a good understanding of the expanded short multiplication method and will begin to represent this as compact short multiplication for TU x U.



Children will be secure in using the grid method for multiplying by two-digit numbers and will begin to explore the links between the grid method and the expanded method of long multiplication.



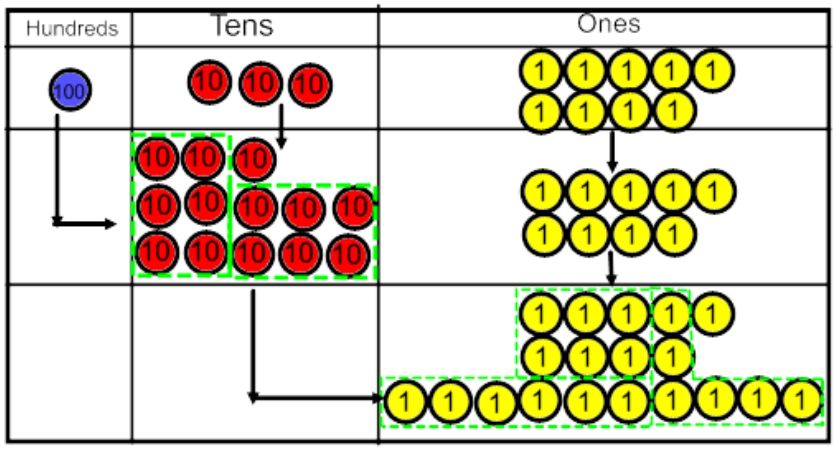
POINTS TO REMEMBER: Use the language '**calculation'** not 'sum' ('Sum' means 'plus' or 'total')
Use the language '**digit'** not 'number' ('Number' is the amount or quantity)
Children should be able to use these strategies independently when problem solving before moving on.

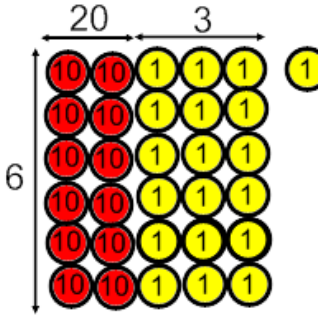
Stage 7 Division

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):
halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ___, ___equal groups

Children will now be secure in using short division for one-digit divisors with an integer quotient.

They will now begin to use the short division notation for calculations involving remainders.





$$\begin{array}{r} 2\ \ 3\ \ r1 \\ 6\,\overline{)\,\cancel{1}\ {}^{1}3\ {}^{1}9} \end{array}$$

Children will also begin to explore the use of jottings of friendly numbers to support long division of calculations with 2-digit divisors.

1 x 15 = 15
2 x 15 = 30 20 x 15 = 300
4 x 15 = 60
8 x 15 = 120
10 x 15 = 150

$$\begin{array}{r} 2\ \ 8 \\ 15\,\overline{)\,4\ \ 2\ \ 0} \\ -\ 3\ \ 0\ \ 0 \quad (20 \times 15) \\ \hline 1\ \ 2\ \ 0 \\ -\ 1\ \ 2\ \ 0 \quad (3 \times 15) \\ \hline 0 \end{array}$$

POINTS TO REMEMBER: Use the language '**calculation'** not 'sum' ('Sum' means 'plus' or 'total')
Use the language '**digit'** not 'number' ('Number' is the amount or quantity)
Children should be able to use these strategies independently when problem solving before moving on.

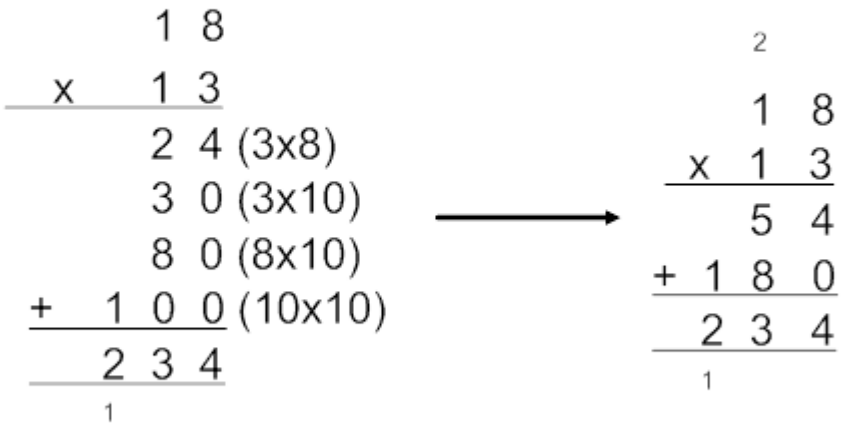
Stage 8 Multiplication

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

counting, steps, each, doubling, scaling, times, twice as big, _____ as big, count in ones, count in ____, lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times, … times, ten times……, times as (big, long, wide…and so on), repeated addition, array, row, column, double, group in pairs, threes…tens, equal groups of, multiplication, product, inverse.

Children will now have a good understanding of the short multiplication method.

Children will now have a good understanding of the expanded long multiplication method and will begin to represent this as compact long multiplication.



Calculating with decimals

When working with decimals, the above stages should always be followed to allow for the development of conceptual understanding. The use of concrete equipment is essential at these stages to secure understanding of the value of each digit in a number (e.g. Place Value Counters, Money). Wherever possible, decimal calculations should be linked to real-life experiences, e.g. money and measures.

POINTS TO REMEMBER: Use the language '**calculation'** not 'sum' ('Sum' means 'plus' or 'total')
Use the language '**digit'** not 'number' ('Number' is the amount or quantity)
Children should be able to use these strategies independently when problem solving before moving on.

Stage 8 Division

Vocabulary (Ensure the correct vocabulary is used at all stages of learning):

halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ___, ___equal groups

Children will now be secure in using short division for one-digit divisors and long division for two-digit divisors with an integer quotient.

They will now explore the use of long division for two-digit divisors that may include a remainder.

The children will begin to interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.

1 x 15 = 15
2 x 15 = 30
4 x 15 = 60
8 x 15 = 120
10 x 15 = 150

20 x 15 = 300

```
      2  8 r12
15 | 4  3  2
   - 3  0  0  (20x15)
     1  3  2
   - 1  2  0  (8x15)
        1  2
```

The answer can also be written as:

$28\frac{12}{15}$ or $28\frac{4}{5}$

Calculating with decimals

When working with decimals, the above stages should always be followed to allow for the development of conceptual understanding. The use of concrete equipment is essential at these stages to secure understanding of the value of each digit in a number (e.g. Place Value Counters, Money). Wherever possible, decimal calculations should be linked to real-life experiences, e.g. money and measures.